Dynamics of Circular Motion

According to Newton's 2nd Law of Motion, the acceleration of an object moving along a circular path must be caused by a net force acting on the object, in the direction of the acceleration.

$$\sum \vec{F} = m \cdot \vec{a}$$
$$= m \cdot \left(\frac{v^2}{r} \right)$$

This force is called the **centripetal force** $\left(\vec{F_c} \right)$ because it is always directed towards the center of the circular path.

$$\boxed{\vec{F_c} = \frac{mv^2}{r}}$$

Note: The centripetal force is not a new and separate force. It is merely a name given to the net force pointing towards the center of the circular path, and is the sum of all the force components that point along the radial direction.

The centripetal force can also be expressed as

$$\boxed{\vec{F_c} = \frac{4\pi^2 mr}{T^2}}$$

by substituting $v = \frac{2\pi r}{T}$.

Solving Problems

When solving circular motion problems, there are several important things you must keep in mind.

1. The equations $v = \frac{2\pi r}{T}$, $\vec{a_c} = \frac{v^2}{r}$, and $\vec{F_c} = \frac{mv^2}{r}$.

2. $\sum \vec{F} = \vec{F_c}$

3. When examining the forces involved in circular motion, you must always ask yourself "What force(s) keeps the object from moving off in a straight line?"

Horizontal Circles

There are numerous examples of situations involving circular motion in a horizontal plane. Some common examples are described below:

1. An object on the end of a string being rotated horizontally (ignoring gravity).

Example 1

A $1.0\ kg$ ball is swung at a constant speed in a horizontal circle of radius $1.2\ m$, on the end of a light string. If the ball has a frequency of $1.0\ Hz$, calculate the tension in the string. Ignore gravity.

2. An object on the end of a string being rotated horizontally (including gravity).

Example 2

A student ties a $0.060\ kg$ lead fishing weight to the end of a piece of string and whirls it around in a horizontal circle. If the radius of the circle is $0.30\ m$ and the object moves with a speed of $2.0\ m/s$, what is the horizontal component of force that directs the lead weight toward the center of the circle? What is the tension in the string?

3. An object making a turn on a level surface.

Example 3

A 1000 kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km/h. What net force is necessary to keep the car moving in a circle around the curve? Will the car make the turn if the pavement is dry and the coefficient of friction is 0.60? What if the road is icy and the coefficient is 0.20?

4. A car turning a corner on a banked road.

Example 4
A $1000\ kg$ car travels around a frictionless, banked curve having a radius of $80\ m$. If the bank is inclined $20°$ to the horizontal, at what speed must the car travel to maintain a constant radius?

5. A rotor ride.

Example 5
In a "rotor ride" at a carnival, people are rotated in a cylindrically walled room. The room radius is $4.6\ m$, and the rotation frequency is 0.50 revolutions per second when the floor drops out. What is the minimum coefficient of static friction so that the people will not slip down?

Circular Motion Worksheet #2

1. It takes a $615\ kg$ racing car $14.3\ s$ to travel at a uniform speed around a circular race track of $50.0\ m$ radius.

   a) What is the acceleration of the car? ($9.65\ m/s^2\ [TTC]$)

   b) What average force must the track exert on the tires to produce this acceleration? ($5937\ N\ [TTC]$)

2. An athlete whirls a $7.00\ kg$ hammer tied to the end of a $1.3\ m$ chain in a horizontal circle. The hammer moves at a rate of $1.0\ revolution/s$. Ignore gravity.

   a) What is the centripetal acceleration of the hammer? ($51.3\ m/s^2\ [TTC]$)

   b) What is the tension in the chain? ($359\ N$)

3. Sue whirls a yo-yo in a horizontal circle. The yo-yo has a mass of $0.20\ kg$ and is attached to a string $0.80\ m$ long. Ignore gravity.

   a) If the yo-yo makes $1.0\ revolution/s$, what force does the string exert on it? ($6.32\ N$)

   b) If Sue increases the speed of the yo-yo to $2.0\ revolutions/s$, what force does the string now exert? ($25.3\ N$)

   c) What is the ratio of answer b to answer a? Why? ($4:1$)

4. A coin is placed on a stereo record revolving at $33\ 1/3$ revolutions per minute.

   a) In what direction is the acceleration of the coin, if any? ($[TTC]$)

   b) Find the acceleration of the coin when it is placed 5.0, 10, and $15\ cm$ from the center of the record. ($0.61\ m/s^2\ [TTC]$, $1.22\ m/s^2\ [TTC]$, $1.83\ m/s^2\ [TTC]$)

   c) What force accelerates the coin?

   d) At which of the three radii listed in b would the coin be most likely to fly off? Why? ($15\ cm$)

5. According to the Guinness Book of World Records, the highest rotary speed ever attained was $2010\ m/s$. The rotating rod was $15.3\ cm$ long. Assume the speed quoted is that of the end of the rod. Ignore gravity.

   a) What is the centripetal acceleration of the end of the rod? ($2.6\times10^{7}\ m/s^{2}\ [TTC]$)

   b) If you were to attach a 1.00 g object to the end of the rod, what force would be needed to hold it on the rod? ($2.6\times10^{4}\ N\ [TTC]$)

   c) What is the period of rotation of the rod? ($4.8\times10^{-4}\ s$)

6. A $0.208\ kg$ toy whistle can be whirled in a horizontal circle of $1.00\ m$ radius at a maximum of $3.00\ rev/s$ before the string breaks. What is the force needed to break the string? $(73.9\ N)$

7. A $0.436\ kg$ ball is suspended on a $0.452\ m$ cord from a fixed point. The ball swings in a horizontal circular path at 0.811 revolutions per second. What is the tension in the cord? $(5.12\ N)$

8. A $0.255\ kg$ ball tethered to a tall pole on a $1.37\ m$ rope is thrown so that it travels in a horizontal circle with the rope making an angle $\theta = 40°$ with the vertical pole.

   a) What is the speed of the ball? $(2.69\ m/s)$

   b) What is the tension in the rope? $(3.26\ N)$

9. A $2.0\ kg$ rock is tied to a light rope $1.0\ m$ long and is swung in a horizontal circle. The rope is at an angle of $30°$ to the horizontal.

   a) What is the tension in the rope? $(39.2\ N)$

   b) What is the speed of the rock? $(3.83\ m/s)$

10. Passengers riding in the Great Six Flags Air Racer are spun around a tall steel tower. At top speed the planes fly at a $56°$ bank approximately $14\ m$ from the tower. In this position the support chains make an angle of $56°$ with the vertical. Calculate the speed of the planes. $(14.3\ m/s)$

11. A carnival rotor ride has a $2.0\ m$ radius and rotates 1.1 times per second.

   a) Find the speed of a rider. ($13.8\ m/s$)

   b) Find the centripetal acceleration of a rider. ($95.5\ m/s^2\ \left[TTC\right]$)

   c) What produces this acceleration?

   d) When the floor drops down, riders are held up by friction. What coefficient of friction is needed to keep the riders from slipping? (0.10)

12. Friction provides the centripetal force necessary for a car to travel around a flat circular race track. What is the maximum speed at which a car can safely travel around a circular track of radius $80.0\ m$ if the coefficient of friction between the tire and the track is 0.30? ($15.3\ m/s$)

13. A curve of radius $120\ m$ is banked at an angle of 18°. At what speed can it be negotiated under icy conditions where friction is negligible? ($19.5\ m/s$)

14. At what angle should a curve of radius $150\ m$ be banked, so cars can travel safely at $25\ m/s$ without relying on friction? ($23°$)

15. A race track curve has a radius of $100\ m$ and is banked at an angle of $68°$. For what speed was the curve designed? ($49\ m/s$)

16. A velodrome track is banked so that a bicycle traveling at $62.3\ km/h$ will have no tendency to slip to either side when traveling on the path that has a radius of curvature of $77.0\ m$. What is the banking angle? ($21.6°$)